

Evolution of perturbations in a domain wall cosmology

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Abstract

A fluid of domain walls has an effective equation of state $p_w = -\frac{2}{3}\rho_w$. This equation of state is qualitatively in agreement with the supernova type Ia observations. We exploit a cosmological model where the matter content is given by a dust fluid and a domain wall fluid. The process of formation of galaxies is essentially preserved. On the other hand, the behaviour of the density contrast in the ordinary fluid is highly altered when domain walls begin to dominate the matter content of the Universe. This domain wall phase occurs at relative recent era, and its possible consequences are discussed, specially concerning the Sachs-Wolfe effect.

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One of the most surprising observational results in cosmology in the end of this century is due to the use of the type Ia Supernova as standard candles in the evaluation of luminosity distance as function of the redshift z . The two groups consacrated to this program [1, 2] arrived at the same conclusion that the Universe is now in an accelerated phase. The inflationary paradigm, initially restricted to the very early Universe, was transferred to the Universe today, with striking consequences, for example, for the age of the Universe and many other cosmological parameters [3, 4]. The most accepted results [5] indicate that the value of the decelerating parameter today is given by $q = -\frac{\ddot{a}a}{\dot{a}^2} \sim -0.66$. If an Universe filled by a perfect fluid is considered, with an equation of state $p = \alpha\rho$, the evolution of the scale factor is given by $a \propto t^{\frac{2}{3(1+\alpha)}}$ and the decelerating parameter reads $q = \frac{1+3\alpha}{2}$. In this case, $q \sim -0.66$ implies $\alpha \sim -0.77$. Hence, the Universe today should be dominated by a fluid with negative pressure such that the strong energy condition is violated.

One of the main issues related to this observational results, is the nature of this negative pressure fluid. The position of the first acoustic peak in the spectrum of the anisotropy of cosmic microwave background radiation is related to the total density of the Universe. In spite of the fact that there is not untill now undoubtfull observational results indicating where precisely this first acoustic peak is located, the recent data coming from BOOMERANG and MAXIMA projects indicates that the density of the Universe is near the critical density [6, 7]. Hence, it can be assumed that the Universe is spatially or nearly spatially flat. On the other hand, the clustered mass is responsible for $0.3 \sim 0.4$ of the critical density. Consequently, from this data it is possible to conclude that $0.7 \sim 0.6$

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of the total matter of the Universe is a smooth component. This smooth component is generally called dark energy.

A fluid of negative pressure violating the strong energy condition does not cluster at large scale. In particular, a cosmological constant, which can be represented by an equation of state $p = -\rho$, remains perfectly smooth, since its density fluctuations are exactly zero. However, the above mentioned results for the deceleration parameter suggest a fluid different from the cosmological constant. A very popular model to describe this dark energy is the so-called "quintessence", a scalar field with an appropriate potential term such that the effective equation of state evolves from a typical radiation equation of state ($p = \frac{\rho}{3}$) to a negative equation of state [8, 9, 10, 11]. But, it is difficult from the available data to exclude others possibilities.

In this work, we will study a "domain wall" cosmological model. Domain walls are topological defects that appears in phase transitions in the early Universe, like others kind of topological defects such as monopoles, cosmic strings and textures. When the topology of the vacuum manifold exhibits disconnected regions we are facing domain walls: this comes out, generally, from a breaking of a discrete symmetry group in the underline field model. Domain walls are characterized by the π_0 homotopy group [12] and its energy-momentum tensor is given by $T_{\mu\nu} = \text{diag}(\rho, 0, \rho, \rho)$. Hence, the equation of state along the y and z spatial components is $p = -\rho$. A network of domain walls may be represented by an isotropic equation of state $p = -\frac{2}{3}\rho$.

From now on, we will not concentrate on a fundamental description of a network of domain walls but, instead, we will consider a phenomenological description of this network. Hence, the main point which will interest us is that this network of domain walls may be described by the above mentioned equation of state which is in the allowed range of possible values determined by the supernova results. There are also claims that the range of allowed values for α may be much more narrow [11]. However, it seems that, in the present state of art, it is not possible to exclude a network of domain walls as one of the possible realizations of the dark energy.

In reference [13] a domain wall dominated Universe has been studied using the so-called solid dark matter model (SDM). In that work, the implications for the anisotropy of cosmic background radiation has been addressed, using some modifications of the CMBFAST code. Some specific signs of such SDM model have being identified. In the present work, we return back to this problem, but trying to develop as far as possible an analytical model. In this way, we intend to identify untill which extent a domain wall phase will affect some observable quantities. Our goal is to control explicitly some physical inputs concerning this domain wall phase.

We will couple a domain wall with a pressureless fluid. Hence, the effective equation of state of this two fluid models evolves from $\alpha = 0$ to $\alpha = -\frac{2}{3}$. In this sense, this phenomenological model exhibits some similarities with the quintessence model. However, the fact that we are exploiting a hydrodynamical description will allow us to find exact solutions. Moreover, we will perform a perturbative study of such model. In such perturbative study, we will allow both fluids to fluctuate (what seems to be the correct way to treat the problem). Then, the consequence of the existence of the domain wall fluid for the evolution of density perturbations will be analysed. In fact, in principle the consequence of the existence of this negative pressure fluid is profound, even with respect to the evolution of density perturbations in the "ordinary" fluid: density contrast in the ordinary fluid stops to increase when the domain wall fluid begins to determine the evolu-

tion of the Universe. But, comparison with some observational data shows that actually the implications of the existence of that "exotic" fluid for the formation of structure and the anisotropy of the cosmic microwave background radiation deserves a much more detailed analysis due to the fact that this fluid dominate the matter content of the Universe quite recently.

We consider a two-fluid model where besides this domain wall component there is also dust whose equation of state is $p_d = 0$. A dust-dominated phase must have occurred prior to the accelerated phase in order to allow gravitational instability to generate local structures. Hence, the field equations are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G \left[T_{\mu\nu}^d + T_{\mu\nu}^w \right] , \quad (1)$$

$$T_{;\mu}^{d\mu\nu} = 0 \quad , \quad T_{;\mu}^{w\mu\nu} = 0 \quad , \quad (2)$$

where $T_{\mu\nu}^d = \rho_d u_\mu u_\nu$ and $T_{\mu\nu}^w = (\rho_w + p_w)u_\mu u_\nu - p_w g_{\mu\nu}$ are the energy-momentum tensor for the dust and domain wall fluids, respectively.

An isotropic and homogenous Universe is represented by the Friedmann-Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - \epsilon r^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] , \quad (3)$$

where $\epsilon = 0, +1, -1$ describe a flat, closed and open spatial section respectively. The corresponding equations of motion are

$$\left(\frac{\dot{a}}{a} \right)^2 + \frac{\epsilon}{a^2} = \frac{8\pi G}{3} (\rho_d + \rho_w) , \quad (4)$$

$$\dot{\rho}_d + 3\frac{\dot{a}}{a}\rho_d = 0 \quad , \quad \dot{\rho}_w + \frac{\dot{a}}{a}\rho_w = 0 \quad (5)$$

Since (5) imply $\rho_d = \frac{\rho_{d0}}{a^3}$ and $\rho_w = \frac{\rho_{w0}}{a}$, there is just one equation to be solved:

$$\left(\frac{a'}{a} \right)^2 = c_1 a + c_2 a^3 , \quad (6)$$

where $c_1 = \frac{8\pi G \rho_{d0}}{3}$, $c_2 = \frac{8\pi G \rho_{w0}}{3}$ and the primes mean derivatives with respect to the conformal time η defined as $dt = a d\eta$. We have fixed also $\epsilon = 0$ (since this value is favoured by observation). Defining $\sinh \theta = \sqrt{\frac{c_1}{c_2}} a$, this non-linear differential equation reduces to

$$\int \frac{d\theta}{\sqrt{\sinh \theta}} = (c_1 c_2)^{1/4} \eta . \quad (7)$$

The final solution for the scale factor is given as

$$a = \sqrt{\frac{c_1}{c_2}} \tan^2 \left(\frac{x(\eta)}{2} \right) , \quad x(\eta) = \text{am} \left((c_1 c_2)^{1/4} \eta \right) , \quad (8)$$

where $\text{am}(z)$ is the Jacobi amplitude function.

The scale factor has two asymptotic regime. For small values of the cosmic time $t \rightarrow 0$, $a \propto t^{2/3} \propto \eta^2$ ($0 < \eta < \infty$); for large values of the cosmic time, $t \rightarrow \infty$, $a \propto t^2 \propto \frac{1}{\eta^2}$

($-\infty < \eta < 0$). The effective equation of state evolves from $p_{eff} \sim 0$ to $p_{eff} \sim -\frac{2}{3}\rho_{eff}$, where $\rho_{eff} = \rho_d + \rho_w$. In fact, the deceleration parameter evolves from $\frac{1}{2}$ to $-\frac{1}{2}$. There is an initial dust dominated phase followed by a domain wall dominated phase.

Since the model developed above display a superluminal expansion for large values of the cosmic time t , an important question is how fluctuations in the ordinary and in the exotic fluid behave. Fluctuations in the dust fluid are affected in two ways: first by the fact that the scale factor changes its behaviour; second, by the fact that fluctuations in the dust are coupled to fluctuations in the domain wall fluid. Since this two-fluid model is coupled through geometry, we will allow all fluids to fluctuate.

As usual, let us introduce small fluctuations around the background solutions found before. In the equations (1,2) it is introduced the quantities $\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$, $\tilde{\rho}_d = \rho_d + \delta\rho_d$, $\tilde{\rho}_w = \rho_w + \delta\rho_w$, $\tilde{u}_d^\mu = u_d^\mu + \delta u_d^\mu$ and $\tilde{u}_w^\mu = u_w^\mu + \delta u_w^\mu$, where in each of these expressions, the right-hand side represents a sum of the background solution and a fluctuation around it. Note that the four-velocity for each fluid may fluctuate independently.

The derivation of the equations which determine the evolution of these quantities is standard [14]. We choose to fix the synchronous coordinate condition $h_{\mu 0} = 0$. In this case, the final equations are, in the conformal time coordinate,

$$h'' + \frac{a'}{a}h' = -\frac{3}{2}\left[2\frac{a''}{a} - 3\left(\frac{a'}{a}\right)^2\right]\Delta_d - \frac{3}{2}\left[2\frac{a''}{a} - \left(\frac{a'}{a}\right)^2\right]\Delta_w \quad , \quad (9)$$

$$\Delta'_d + \Psi - \frac{h'}{2} = 0 \quad , \quad (10)$$

$$\Delta'_w + \Theta - \frac{h'}{2} = 0 \quad , \quad (11)$$

$$\Psi' + \frac{a'}{a}\Psi = 0 \quad , \quad (12)$$

$$\Theta' + 3\frac{a'}{a}\Theta + 2n^2\Delta_w = 0 \quad . \quad (13)$$

In deriving these equations we have made the following redefinitions: $h = \frac{h_{kk}}{a^2}$, $\Delta_d = \frac{\delta\rho_d}{\rho_d}$, $\Delta_w = \frac{\delta\rho_w}{\rho_w}$, $\Psi = a\delta u_{d,i}^i$, $\Theta = a\delta u_{w,i}^i$. Moreover, the spatial dependence of each quantity is such that the Helmholtz equations is obeyed: $\nabla^2 Q(\vec{x}, t) = -n^2 Q(\vec{x}, t)$

Equations (9,10,11,12,13) seems to admit no exact solution, not only because of the coupling between all quantities, but mainly because of the complicated form of the background expression for the scale factor. However, it is possible to obtain analytical solutions in the asymptotic regimes. Before to determine these analytical solutions, we remark that it is possible to set $\Psi = 0$, since this quantity decouples from the others, and it contribute only through a decreasing inhomogenous term. Hence, we can reduce the above system of equations to just two coupled equations:

$$\Delta''_d + 3\frac{a'}{a}\Delta'_d = 3\Delta''_w + 9\frac{a'}{a}\Delta'_w - 2n^2\Delta_w \quad , \quad (14)$$

$$\Delta''_d + \frac{a'}{a}\Delta'_d + \frac{3}{4}\left[2\frac{a''}{a} - 3\left(\frac{a'}{a}\right)^2\right]\Delta_d = -\frac{3}{4}\left[2\frac{a''}{a} - \left(\frac{a'}{a}\right)^2\right]\Delta_w \quad . \quad (15)$$

First, let us solve the perturbed equations for the dust phase. In this case, $a \propto \eta^2$. and equation (15) reduces to

$$\Delta''_d + 2\frac{\Delta'_d}{\eta} - 6\frac{\Delta_d}{\eta^2} = 0 \quad (16)$$

leading to the well known solution for the evolution of the density contrast in a pure dust Universe: $\Delta_d \propto \eta^2$. Hence, in the beginning the domain wall fluid do not influence either the background and the perturbed quantities. We can solve also the homogenous equation for the fluctuation in the domain wall fluid, obtaining

$$\Delta_w = \eta^{-5/2} \left\{ C_1 I_{5/2}(\sqrt{\frac{2}{3}} n \eta) + C_2 K_{5/2}(\sqrt{\frac{2}{3}} n \eta) \right\} + C_3 \quad (17)$$

where $K_\nu(x)$ and $I_\nu(x)$ are the modified Bessel's functions and the C_i are integration constants.

In the other asymptotic, the domain wall fluid dominate and the scale factor behaves as $a \propto \eta^{-2}$, where $t \rightarrow \infty$ means $\eta \rightarrow 0_-$. The coupled system (14,15) reduces to

$$\Delta_d'' - 6 \frac{\Delta_d}{\eta} = 3\Delta_w'' - 18 \frac{\Delta_w'}{\eta} - 2n^2 \Delta_w, \quad (18)$$

$$\Delta_d'' - 2 \frac{\Delta_d'}{\eta} = -6 \frac{\Delta_c}{\eta^2}, \quad (19)$$

which can be expressed in terms of a single third order equation:

$$\Delta_w''' - 7 \frac{\Delta_w}{\eta} + \left[-\frac{2}{3} n^2 + \frac{14}{\eta^2} \right] \Delta_w' + \left[\frac{2}{3} \frac{n^2}{\eta} - \frac{14}{\eta^3} \right] \Delta_w = 0. \quad (20)$$

This equation can be solved remembering that the synchronuous coordinate condition has a residual coordinate freedom [15]. Using this fact, it is easy to see that $\Delta_w \propto \eta$ is a solution of the third order differential equation. Hence, it is possible to reduce the order of the equation obtaining the final solution

$$\Delta_w = \eta \left\{ \int \eta^{\frac{5}{2}} \left[D_1 I_{5/2}(\sqrt{\frac{2}{3}} n \eta) + D_2 K_{5/2}(\sqrt{\frac{2}{3}} n \eta) \right] + D_3 \right\}, \quad (21)$$

where the D_i are again integration constants.

We are now able to analyse the results obtained. The first important point is that in the beginning of the dust dominated phase, the domain wall fluid plays no important role: the dust density contrast evolves exactly as in a pure dust model. Since, it is at this period that galaxies form, it is possible to state that the presence of the domain wall fluid will not spoil this scenario. The density contrast for the domain wall fluid exhibits, on the other hand, features that deserves some comments. In the long wavelength limit $n \rightarrow 0$, it presents a constant mode and a decreasing mode. However, in the small wavelength limit $n \rightarrow \infty$, it displays an exponential increasing mode. This may lead to instabilities. However, it has been shown in [16, 17] that these instabilities are consequence of the hydrodynamical approach employed here. The hydrodynamical approach is a phenomenological description, which mimics some features of this fluid of topological defects. However, when we substitute this hydrodynamical approach by a field description, the instabilities in the small wavelength limit disappears, keeping the behaviour in the long wavelength limit unaltered. It has been showed in [17] that the long wavelength limit is insensitive to the approach used.

In the other asymptotic regime, the domain wall fluid exhibits only decreasing modes. In fact, when $n \rightarrow 0$, we find

$$\Delta_w \sim D_1 \eta^6 + D_2 \eta^2 + D_3 \eta \quad (22)$$

which goes to zero as time evolves. In this same limit, the density contrast in the dust fluid exhibits decreasing and constant modes:

$$\Delta_d \sim D_1 \eta^7 + D_2 \eta^2 + D_3 \quad . \quad (23)$$

Hence, as the domain wall fluid dominates the matter content of the Universe, density perturbations in the ordinary fluid do not grow anymore. This is essentially due to the coupling of both fluids at perturbative level.

The main question that comes out from these results is if there is other observational consequences. In fact, this is a much more difficult question for the following reasons. Observations today indicate that around 40% of the matter of the Universe suffers the process of gravitational collapse, while the others 60% remain a smooth component. Accepting that the Universe is flat, we have than $\Omega_c \sim 0.4$ and $\Omega_s \sim 0.6$ today, where the subscripts designates the clustered and the smooth components of the Universe. The clustered component may be not baryonic but it is quite possibly a cold component, i.e., a pressureless fluid, like the ordinary matter introduced in the model developed above. The smooth component must violate the strong energy condition, as the domain wall fluid considered here. Since $\Omega_c \propto a^{-3}$ and $\Omega_s \propto a^{-1}$, fixing the value of the scale factor equal to one today, we can evaluate when the smooth component begins to dominate the matter content of the Universe. Imposing $\Omega_c = \Omega_s$ and remembering that the redshift is given by $z = -1 + \frac{a_0}{a}$, we find that the domain wall fluid dominated era begins at $z \sim 0.22$. Accepting that the age of the Universe is $t_0 \sim 13 \text{ Gy}$, this happens at $t \sim 11.7 \text{ Gy}$. This is quite recent.

Of course, this domination of the domain wall fluid drives an accelerated expansion, which is reflected in the high redshift supernova measurements. For the anisotropy of the cosmic microwave background radiation the situation is much less clear. If we think on the integrated Sachs Wolfe effect, the domain wall dominate at about 10% of the integration interval. But we must be cautious in saying that this would lead to no modification at all with respect to a pure dust Universe since we must verify how different multipole moments are affected during the travel from the last scattering surface to the observer today. It may happen that for some class of multipole moments this modification at the very end of the trajectory leads to a quite different behaviour.

However, even qualitatively it is possible to verify that the domain wall dominated phase will have some consequences for the anisotropy of the cosmic microwave background radiation. This anisotropy is determined by the Sachs-Wolfe formula

$$\frac{\Delta T}{T} = \frac{1}{2} \int_{\eta_e}^{\eta_r} \frac{d}{d\eta} \left(\frac{h_{ij}}{a^2} \right) e^i e^j d\eta \quad , \quad (24)$$

where we have tried to keep our previous definitions; η_e and η_r are the emission and reception time and e^i is an unitary vector defining the direction of observation. The perturbed metric h_{ij} may be decomposed conveniently as

$$h_{ij} = a^2 \left(h_1 \delta_{ij} Q + \frac{h_2}{n^2} Q_{,i,j} \right) \quad . \quad (25)$$

Hence, $h = 3h_1 + h_2$. It is possible to write down equations governing the behaviour of these metric functions [18]. Transposing these equations for our definitions, we obtain in particular

$$h_1'' + 2 \frac{a'}{a} h_1' = - \left[2 \frac{a''}{a} - \left(\frac{a'}{a} \right)^2 \right] \Delta_w \quad . \quad (26)$$

In the long wavelength limit this equation leads to

$$h'_1 \sim D_1 \eta^5 + D_2 \eta + D_3 + D_4 \eta^4 \quad , \quad (27)$$

with a similar expression for h'_2 . Hence, a constant mode will appear in the integral of the Sachs-Wolfe effect. In spite of the fact that density perturbations remain constant or decrease, we must expect important distortions in the cosmic microwave background radiation. It is important to note that the constant mode is associated to the residual coordinate freedom: but this mode may have physical meaning due to the junction conditions between the different phases of evolution of the Universe.

Domain walls are topological defects, metastable, which could be originated in phase transitions in the primordial Universe. It has already be argued that domain walls may play an important role in the evolution of the Universe [12]. Indeed, domain walls may lead to an accelerated Universe, with a value for the decelerating parameter compatible with the results coming from supernova type Ia observations. We have verified here that the presence of this fluid does not spoil the formation of galaxies process at the begining of the dust dominated phase. However, in the deep domain wall phase, matter perturbations do not grow anymore due to the coupling with the domain wall fluid. Even if the domain wall fluid would dominate for a small interval of time, this may have important consequences.

The goal of the present paper was to develop an analytical domain wall cosmological model, verifying in particular its consequence for the evolution of density perturbations and for the Sachs-Wolfe effect. In a preceding work [19], we have studied a two-fluid model where ordinary matter was coupled to a cosmic string fluid. A network of cosmic string may be represented by an equation of state $p = -\frac{\rho}{3}$. Hence, in principle such fluid is outside the observational limits imposed by supernova type Ia observations, in opposition to what happens in the case of domain walls. But, there are other important differences: when the string fluid dominates the evolution of the Universe, perturbations in the ordinary fluid may still grow, although very slowly. In the case of domain walls, perturbations in the ordinary fluid simply ceases to grow. In some sense, this is due to the fact that in the case of cosmic string, density contrast in the cosmic string fluid decouples from density contrast in the ordinary fluid, what does not happen in the case of domain walls. As it has been argued before, this may lead also to consequences in what concerns the anisotropy of cosmic microwave background. We intend to perform such analysis from the fundamental definition of the Sachs-Wolfe effect in terms of the metric functions.

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References

- [1] A.G. Riess et al, *Astron. J.* **116**, 1009(1998);
- [2] S. Permultter et al, *Astrophys. J.* **517**, 565(1998);
- [3] J.R. Primack, *Cosmological parameters*, astro-ph/0007187;
- [4] B. Novosyadlyj, R. Durrer and S. Apunevych, *Cosmological parameters from observational data on the large scale structure of the Universe*, astro-ph/0009485;

- [5] G. Efstathiou, *Constraining the equation of state of the Universe from distant type Ia supernovae and cosmic microwave background anisotropies*, astro-ph/9904356;
- [6] W. Hu, M. Fukugita, M. Zaldarriaga and M. Tegmark *CMB observables and their cosmological implications*, astro-ph/0006436;
- [7] A.E. Lange et al., *First estimations of cosmological parameters from BOOMERANG*, astro-ph/0005004;
- [8] L. Wang, R.R. Caldwell, J.P. Ostriker and P.J. Steinhardt, *Astrophys. J.* **530**, 17(2000);
- [9] S. Perlmutter, M.S. Turner and M. White, *Phys. Rev. Lett.* **83**, 670(1999);
- [10] I. Zlatev, L. Wang and P.J. Steinhardt, *Phys. Rev. Lett.* **82**, 896(1999);
- [11] Ph. Brax and J. Martin, *Phys. Lett.* **B468**, 40(1999);
- [12] A. Vilenkin and E.P.S. Shellard, **Cosmic strings and other topological defects**, Cambridge University Press, Cambridge (1994);
- [13] R.A. Battye, M. Bucher and D. Spergel, *Domain wall dominated universes*, astro-ph/9908047;
- [14] S. Weinberg, **Gravitation and cosmology**, Wiley, New York (1972);
- [15] P.J.E. Peebles, **The large scale structure of the Universe**, Princeton University Press, Princeton (1980);
- [16] J.C. Fabris and J. Martin, *Phys. Rev.* **D55**, 5205(1997);
- [17] J.C. Fabris, S.V.b. Gonçalves and N.A. Tomimura, *Class. Quant. Grav.* **17**, 2983(2000);
- [18] L.P. Grishchuk, *Phys. Rev.* **D50**, 7154 (1994).
- [19] J.C. Fabris and S.V.B. Gonçalves, *Phys. Rev.* **D56**, 6128(1997).